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Core Knowledge

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Compex cognitive skills such as reading and calculation and complex cognitive achievements such as formal science and mathematics may depend on a set of building block systems that emerge early in human ontogeny and phylogeny. These core knowledge systems show characteristic limits of domain and task specificity: Each serves to represent a particular class of entities for a particular set of purposes. By combining representations from these systems, however, human cognition may achieve extraordinary flexibility. Studies of cognition in human infants and in nonhuman primates therefore may contribute to understanding unique features of human knowledge.

How do humans develop and deploy complex, species-specific, and culture-specific cognitive skills such as reading, mathematics, mapmaking, myriad forms of tool use, and reasoning about the physical and social world? Studies of adults who have mastered these skills, of children who are in the process of learning them, and of people who experience specific difficulties in skill acquisition or use have long been undertaken to probe these abilities. My goal today is to suggest that we broaden our search for insights into complex cognitive skills by considering the findings of research on two additional populations: very young children, who have not yet begun the process of skill acquisition, and nonhuman animals, who are destined never to acquire them. My reasoning is simple: When children and adults construct new cognitive abilities, they build on component cognitive systems with a long ontogenetic and phylogenetic history. Studies of infants and of nonhuman primates can shed light on these *core knowledge* systems.

Editor's Note
Elizabeth S. Spelke received the Award for Distinguished Scientific Contributions. Award winners are invited to deliver an award address at APA's annual convention. This award address was delivered at the 108th annual meeting, held August 4–8, 2000, in Washington, DC. Articles based on award addresses are not peer reviewed, as they are the expression of the winner's reflections on the occasion of receiving an award.

What are core knowledge systems? Studies of human infants suggest that they are mechanisms for representing and reasoning about particular kinds of ecologically important entities and events—including inanimate, manipulable objects and their motions, persons and their actions, places in the continuous spatial layout and their Euclidean geometric relations, and numerosities and numerical relationships. These systems serve to build representations of objects, persons, places, and numerosities that encompass quite abstract properties and relationships, such as the persistence of objects over occlusion and the goals of perceivable acts. Infants' core systems appear to be very similar to those of many nonhuman animals, suggesting that they have a long evolutionary history. Nevertheless, core knowledge systems are limited in a number of ways: They are *domain specific* (each system represents only a small subset of the things and events that infants perceive), *task specific* (each system functions to solve a limited set problems), and *encapsulated* (each system operates with a fair degree of independence from other cognitive systems).

Research on older children and on adults suggests that the core knowledge systems found in infants contribute to later cognitive functioning in two ways. First, core systems continue to exist in older children and adults, giving rise to domain-specific, task-specific, and encapsulated representations like those found in infants. Second, core systems serve as building blocks for the development of new cognitive skills. When children or adults develop new abilities to use tools, to perform symbolic arithmetic calculations, to read, to navigate by maps and landmarks, or to reason about other people's mental states, they do so in large part by assembling in new ways the representations delivered by their core systems.

To make this picture of human cognition and cognitive development more concrete, I focus here on a single case study of core knowledge and cognitive development, centering on the domain of number. My story begins with two core knowledge systems found in human infants and in nonhuman primates: a system for representing objects and their persistence through time and a system for representing approximate numerosities. Then I ask how young children may build on these two systems to learn verbal counting and to construct the first natural number concepts. Finally, I consider how the same systems may contribute to mathematical thinking in adults.

Core Knowledge of Objects

Twenty years of research provides evidence that infants build representations of objects as complete, connected, solid bodies that persist over occlusion and maintain their identity through time (e.g., Baillargeon, 1993; Spelke & Van de Walle, 1993). One of the situations that reveal this ability was devised by Karen Wynn (1992). Wynn's studies used a preferential looking–expectancy violation method, based on the assumption that infants would look longer at an unexpected event than at an otherwise similar but expected one. In one experiment, 5-month-old infants saw a single toy animal placed on a stage, a screen was lowered to occlude the toy, a second, featurally identical toy was introduced in view of the child, and then that toy was placed behind the screen. Finally, the screen was removed to reveal either one or two toys on alternating trials. If infants failed to keep track of these objects over occlusion, then they might be expected to look longer at the display of two toys, because only one toy ever had been visible at a time. If infants kept track of each object as it moved behind the occluder and maintained distinct representations of the two objects, then the display containing just one toy would have been unexpected, eliciting longer looking. The latter looking preference was obtained. In subsequent studies, moreover, infants presented with the task of adding one object to another looked longer at three objects than at two objects, indicating that their representation of two objects was exact, and infants presented first with an array of two objects and then with the removal of one object from behind the occluder successfully computed the subtraction of two minus one to yield one object rather than two (Wynn, 1992).

Wynn's (1992) exciting findings generated many replications and extensions. In particular, Simon, Hespos, and Rochat (1995) found that infants responded appropriately to the number of objects in Wynn's task even when the features of those objects changed behind the occluder (e.g., when an Elmo puppet was replaced by an Ernie puppet), indicating that infants truly were representing the number of objects and not the amount of some property common to the objects such as their coloring or detailed shape. Moreover, Koechlin, Dehaene, and Mehler (1998) found that infants responded to number in Wynn's task even when the occluded objects moved on a turntable, so that their locations were variable and unpredictable, indicating that infants responded to object number rather than to object locations. As Wynn (1992) observed, infants' looking preferences in these experiments provided evidence for three kinds of representations: representations of objects as enduring bodies over occlusion, representations of number—of the distinction between one, two, and three objects, and representations of the operations of addition and subtraction of one object.

All of the previously discussed studies used a preferential looking method, and so one may ask whether the competencies they reveal are specific to that method. This question is difficult to answer for 5-month-old infants, because their action systems are so limited, but further studies, focusing on 8- to 12-month-old infants, provide evidence for the same abilities using two quite different response systems: manual search and locomotion. In the box search task, Van de Walle, Carey, and Prevor (in press) presented Wynn's (1992) one-plus-one event by successively hiding two objects in a box, surreptitiously removing one object from the box, allowing infants to retrieve the one remaining object and then taking it away, and observing infants' further exploration of the box. Relative to infants who originally saw just one object hidden in the box, infants who had seen the one-plus-one event searched the box longer and more persistently, as if they expected to find a second object. In the locomotor choice task, Feigenson, Carey, and Hauser (2000) presented infants with two boxes into which they placed different numbers of cookies, one at a time. After two cookies were placed in one box and three cookies in the other box, the boxes were widely separated, and infants were allowed to crawl toward them. Infants selectively approached the box with the three cookies, suggesting that they represented the numbers of cookies in the boxes.

Three tasks using different response systems and different objects therefore provide evidence that human infants keep track of objects that become occluded and construct, from a pattern of successive occlusion, a representation of the precise number of objects in the array. Further studies reveal, however, two interesting limits to infants' abilities. First, infants fail to represent number when presented with entities that do not behave as objects do. For example, Huntley-Fenner and Carey (2000) repeated Wynn's (1992) studies using nonsolid, noncohesive sandpiles instead of objects, and they found no consistent response to the number of sandpiles. As a second example, Chiang and Wynn (in press) conducted experiments similar to Wynn's using collections of objects: a pile of blocks. Except in cases where the collections could be represented as separate, individual objects, infants failed to track the collections over occlusion. These findings provide evidence that the system of representation at work in this family of experiments is domain specific: It operates on objects but not on other detectable entities.

The second limitation to infants' object representations in Wynn's task appears as the number of objects is increased. Infants succeed in representing objects over occlusion as long as the total number of objects behind an occluder is small—up to about three—but they fail with larger numbers. For example, Feigenson et al.'s (2000) participants in the locomotor search experiments successfully approached a box containing two cookies rather than one or three cookies rather than two, but they failed to approach a box containing eight cookies rather than four. Although infants can keep track of multiple objects over occlusion, this ability appears to break down as the number of objects increases beyond about three.

In summary, diverse findings provide evidence that infants have a system for representing objects that allows them to keep track of multiple objects simultaneously. The system is domain specific (it applies to objects but not to other perceptible entities such as sandpiles), it is subject to a set size limit (it allows infants to keep track of about three objects but not more), and it survives changes in a number of object properties, including color, detailed shape, and spatial location. The same system appears to exist in a population of nonhuman primates—adult, semi-free-ranging rhesus monkeys—that has been tested with the same methods and the same types of stimuli as human infants. Like infants, monkeys can add one object to another to form a representation of two objects when given Wynn's preferential looking task (Hauser, MacNeilage, & Ware, 1996). Monkeys also use such representations to guide their search through a single box or their choice between two boxes when tested with the box search and locomotor choice tasks (Hauser, Carey, & Hauser, 2000). Like human infants, adult monkeys show a set size limit in their number representations: In the locomotor choice task, for example, they consistently choose the box with more objects when given a choice of one versus two, two versus three, or three versus four, but they fail with a choice of four versus eight. Human infants and adult monkeys form very similar core representations of objects.

Does this system of representation also exist in human adults? Scholl and Leslie (1999) have noted that the infant's system of object representation shows some intriguing similarities to a system of representation that is used by adults when they have to follow visible objects over time and make rapid decisions about them. One task Scholl and Leslie have considered is the multiple-object-tracking task of Pylyshyn and Storm (1988). Participants are presented with an array of identical visible forms, say, eight white circles. Initially the array is stationary, a subset of target circles is briefly flashed, and then all the circles begin to move continuously and independently. During this motion, participants must attend to the targets that flashed at the beginning and indicate whether a single circle that flashes at the end of the trial is one of them. This task is very easy when participants attend to one or two targets, it becomes more difficult when they must attend to three or four targets at once, and it becomes nearly impossible when they must attend to larger numbers of targets. Like infants and monkeys, adults can track multiple objects but show a set size limit. Whereas the human infant's limit is about three objects, the adult's limit, like the adult monkey's, is about four.

Wynn's (1992) task with infants and Pylyshyn and Storm's (1988) task with adults show some similarities, but is the same core knowledge system at work in both cases? A beautiful series of experiments by Brian Scholl suggests that it is. Scholl tested adults on the multiple-object-tracking task to see whether they succeeded, and failed to track objects under the same conditions as infants. One study, for example, compared adults' multiple-object tracking under conditions in which objects appeared either to become temporarily occluded or to temporarily go out of existence (*implosion;* (Scholl & Pylyshyn, 1999). In studies dating back to Bower (1966) and others, infants have been found to track objects over occlusion but not over implosion. Like infants, Scholl and Pylyshyn's adults tracked successfully in the occlusion event but not in the implosion event. In a second study, Scholl and Pylyshyn asked whether adults would succeed in tracking objects if features of the objects such as color and shape changed during the tracking task. Like infants in Simon et al.'s (1995) studies, adults were unaffected by these feature changes. In a third study, Scholl, Pylyshyn and Feldman (in press) asked whether adults would lose their ability to track separate objects if the objects lost their boundaries: a limit suggested by studies of infants (Spelke & Van de Walle, 1993). The same eight circles were presented for tracking, but pairs of circles now were connected by lines, so that there no longer appeared to be eight separate objects but four connected "barbells." When participants were asked to track four of the eight ends of the barbells, they failed utterly to do so. Like that of infants, adults' system of object representation is domain specific: It serves to keep track of spatially distinct, bounded objects but not of other perceptible entities, such as spatially connected object parts.

Scholl and Pylyshyn's (1999) and Scholl, Pylyshyn, and Feldman's (in press) experiments illustrate how one can turn an apparent similarity between cognitive functioning in infants and adults into a testable set of hypotheses probing whether a single core system is at work at the two ages. In this case, all the evidence so far suggests that the same cognitive system underlies infants' representations of objects in addition tasks and adults' representations of objects in multiple-object-tracking tasks. This system builds representations of objects that survive occlusion and that adults are consciously aware of, but it is domain specific (it applies to objects but not to parts of objects) and task specific (it supports object tracking over occlusion but not over implosion). Moreover, the internal workings of this system are opaque to adults and at odds with some of their beliefs. Adults believe, for example, that objects do not spontaneously undergo radical changes in shape and color while they are hidden, yet the system of representation that guides adults' object tracking is impervious to these changes. These are hallmarks of a core knowledge system.

Core Knowledge of Numerosity

I turn now to a second core knowledge system that serves to represent approximate numerical magnitudes. Many studies of number representation, both in infants and in adults, have been plagued by a tricky methodological problem: Whenever two displays differ in numerosity, they differ on other, continuous dimensions as well. For example, if two sets with different numbers of objects present objects of the same sizes and colors, then the more numerous set also will present a larger colored surface area. And if the sets present objects that appear at equal densities, then the more numerous set will cover a larger region of the display. Recent experiments by Fei Xu, Jennifer Lipton, and Hilary Barth nevertheless have circumvented these problems and provide a test of infants' and adults' representations of large numerosities.

Xu's experiments used a different preferential looking method, focusing on infants' tendency to look longer at novel arrays than at more familiar ones. In Xu and Spelke (2000b), 6-month-old infants were presented with a succession of arrays of dots on a series of familiarization trials. From trial to trial, the positions and sizes of the dots changed, but the number of dots remained the same: 8 dots for half the participants and 16 dots for the rest. Moreover, the arrays of 8 and 16 dots were equated for overall size (and therefore differed in density) and for overall brightness and covered surface area (and therefore differed in average element size). After looking time to the array sequence had declined, all the infants were presented with test arrays of 8 versus 16 dots on alternating trials. For the test, arrays at the two numerosities were equated for density and element size and differed, therefore, in overall size, brightness, and total filled surface area. These stimulus controls effectively disentangled responses to number from responses to correlated continuous variables: If infants failed to respond to number and instead responded to variables like density or brightness, then the infants in the two groups would have looked equally at the two test displays.

In this experiment, infants looked longer at the display presenting the novel numerosity, providing evidence that they discriminated between the 8- and 16-dot arrays on the basis of their numerosity. Xu and Spelke (2000a) replicated this effect with larger numerosities: Six-month-old infants successfully discriminated 16 from 32 dots. In contrast, infants in Xu and Spelke's (2000a, 2000b) experiments failed to discriminate 8 from 12 or 16 from 24 dots when tested by the same method, just as infants in an earlier experiment by Starkey and Cooper (1980), tested with a similar method although without the same stimulus controls, failed to discriminate 4 from 6 dots. These findings suggest that infants' large-number discriminations are imprecise and depend on the ratio of the set sizes to be discriminated: Infants succeed with set sizes in a 2:1 ratio such as 8 versus 16 or 16 versus 32, but they fail with set sizes in a 3:2 ratio such as 8 versus 12 or 4 versus 6.

Very recent studies by Lipton and Spelke (2000) have asked whether infants' sensitivity to numerosity is robust enough to appear when infants are tested with different types of stimuli and with a different behavioral response. Lipton and Spelke used the head-turn preference procedure developed for studies of speech perception in infancy (Jusczyk, 1997) to test 6-month-old infants' sensitivity to numerosity in sequences of sounds. On a series of familiarization trials, infants heard different sequences of sounds played through a speaker located to their left or right, and the time they spent with their head turned toward the speaker was measured after the offset of each sequence. On different trials, the individual sounds differed in duration, spacing, and quality (six different synthesized sounds were used in all), but they always presented the same numerosity: 8 for half the infants and 16 for the others. After head orientation to the sounds declined, infants in both groups were presented with new sequences of 8 and 16 sounds in alternation. During familiarization and test, the durations of individual sounds and the total sequence durations were varied, as in Xu and Spelke's (2000a, 2000b) experiments, to dissociate responses to number from responses to these continuous variables. Infants oriented toward the speaker for a longer duration after hearing the novel numerosity than after hearing the familiar numerosity, providing evidence that they discriminated between the 8- and 16-item sequences on the basis of numerosity. In a follow-up study, infants failed to discriminate 8- from 12-item sequences, as they had failed with spatial arrays of dots. These findings provide further evidence that infants' representation of large numerosities is imprecise and that numerical discrimination depends on the ratio of set sizes. Intriguingly, they suggest the same ratio limits apply to visual, spatial arrays of dots and to auditory, temporal arrays of sounds. These findings provide an initial hint that representations of large, approximate numerosities may be independent of sensory modality or stimulus format (temporal vs. spatial).

Aside from the set size ratio limit, experiments on infants suggest two other limits to infants' abilities to discriminate between large numerosities. First, we have already noted that in locomotor choice tasks, infants are unable to discriminate between large numbers of objects when the objects must be tracked individually over occlusion. When four cookies are placed successively in one box and eight cookies are placed successively in another box, infants fail to respond to this numerical difference, even though the numerosities differ by a 2:1 ratio. A second difference was revealed when Xu and Spelke (2000a) repeated their dot-discrimination experiments with small numbers of dots: six-month-old infants failed to discriminate between arrays of one versus two dots, or two versus three dots, when discrimination was tested using the same controls for continuous variables that Xu and Spelke (2000a) used for the larger numerosities. This finding initially surprised us, but it is a robust one, observed in independent experiments by Clearfield and Mix (1999) with two-dimensional patterns and by Feigenson, Carey, and Spelke (2000) with three-dimensional objects. Although infants treat large numbers of visible items as a set and they discriminate between these sets on the basis of their numerosity, they appear to treat small numbers of visible items as individual objects that can be tracked over time but not as a set with a specific cardinal value that can be instantiated by different objects at different times.

What mechanism underlies infants' representations of large numerosities, and how does it relate to the mechanism that underlies infants' representations of small numbers of objects in Wynn's (1992) addition and subtraction tasks? Because both the infants in Wynn's (1992) tasks and those in Xu and Spelke's (2000a, 2000b) and Lipton and Spelke's (2000) tasks respond to the number of individuals in an array, a natural hypothesis is that the same system of representation of numerosity underlies performance both with small numerosities and with large ones. This hypothesis has gained many supporters (e.g., Dehaene, 1997; Wynn, 1998), but I think the weight of the evidence goes against it. Three sets of findings suggest that the system that underlies representations of small numbers of objects is different from the system that underlies representations of large numerosities. First, performance with small and large numbers is subject to different limits: Wynn's small-number task shows a set size limit of three, whereas Xu and Spelke's (2000a, 2000b) large-number tasks show a set size ratio limit of 2:1. Second, performance with small numbers of objects is robust over occlusion, but performance with large numbers of objects is not: Infants can discriminate large set sizes when the elements are continuously available but not when they appear and are occluded one at a time. Third, performance with large numbers of items is robust over variations in continuous quantities including item size, total surface area, density, and array size, but performance with small numbers of items is not: Infants fail to discriminate one from two dots or objects when continuous quantities are strictly equated across the arrays.

Putting all these findings together, I suggest that two core knowledge systems are at work in these experiments. One is the system for representing objects and their persisting identity over time, as already described. The other is a system for representing sets and their approximate numerical values. These systems are domain specific (one applies to objects, the other to sets), task specific (one allows for addition of one, the other allows for comparisons of sets), and independent (the situations that evoke one are different from the situations that evoke the other).

I have discussed evidence that the core object system found in infants also exists in adult monkeys; what about the core numerosity system? A wealth of research provides evidence for representations of large, approximate numerosities in many nonhuman animals, including rats and pigeons as well as primates (see Boysen & Capaldi, 1993, and Gallistel, 1990, for reviews). For nonhuman animals as for infants, discrimination depends on the ratio of the numerosities to be discriminated and is otherwise independent of set size. In many situations, animals form abstract representations of numerosity that survive changes in the modality or format of presentation (see Gallistel, 1990). Finally, monkeys fail to discriminate between large, approximate numbers of objects when the objects are introduced and are occluded one by one (Hauser et al., 2000). These findings suggest that the core representations of numerosity found in human infants are similar to those of various other animals.

Much research also provides evidence that human adults can represent large approximate numerosities when presented with spatial arrays of dots or sequences of sounds. Do these abilities depend on the same mechanisms as the abilities found by Xu and Spelke (2000a, 2000b) and Lipton and Spelke (2000) in infants? A new series of experiments (Barth, Kanwisher, & Spelke, 2000) suggests that they might. First, Barth et al. asked whether adults, like infants, could discriminate between two visual arrays of dots, two visual sequences of light flashes, or two temporal sequences of sounds on the basis of their numerosity, when continuous variables such as array brightness, element density, or sequence duration were controlled. Adults were found to have these abilities. Next, Barth et al. asked whether for adults, as for infants, the ability to discriminate between two arrays or sequences with large numbers of elements depended on the ratio of the two set sizes. Adults' numerosity discrimination was tested at a range of set sizes (from 10 to about 70 items in different experiments) and set size ratios (1:3 to 6:7). Performance depended only on set size ratio: Discriminating 40 from 60 dots was as easy as discriminating 20 from 30 dots and easier than discriminating 40 from 50 dots. The only difference between adults and infants concerned the precision of numerosity representations: Although infants failed to discriminate between sets in a 3:2 ratio, adults easily succeeded at this ratio and performed above chance even at the highest ratio tested.

Barth et al.'s (2000) final experiment was inspired by studies of numerical discrimination in nonhuman animals. They asked whether adults' numerosity discrimination depended on representations that were abstract and independent of format or sensory modality. Adults were presented with three types of displays: visual spatial arrays of dots, visual temporal sequences of light flashes, and auditory temporal sequences of sounds. On some blocks of trials, adults compared the numerosities of two arrays with a common modality and format, as in the previous studies. In other blocks of trials, adults compared numerosities across modalities, formats, or both. In the latter case, participants had to judge whether a spatial array of dots had more or fewer elements than a temporal sequence of sounds. Unanimously, adults predicted that the tasks with heterogeneous arrays would be more difficult than the tasks with homogeneous arrays, and many adults expressed low confidence in their judgments. The accuracy of their numerical comparisons, however, was just as high for the heterogeneous arrays as for the homogeneous arrays. These findings suggest that human adults have a system for representing large, approximate numerosities independently of modality or format but that the system is so encapsulated that most of us don't even believe we have it! Comparisons with infants suggest that it is a core knowledge system that emerges early in infancy, increases in precision over development, and persists throughout life.

So far, I have suggested that human infants, various nonhuman animals, and human adults have two core knowledge systems. What role do these systems play in the development of complex cognitive skills? To approach this question, I consider how children develop the number concepts that are at the heart of the elementary school curriculum: concepts of the natural numbers and of simple arithmetic.

Learning of Number Words and the Counting Routine

Research by many investigators provides evidence that before most children get to school, they have a basic understanding of the natural numbers (Butterworth, 1999, Gelman & Gallistel, 1978). Children understand, for example, that numbers form a progression that starts with one and continues by successive additions of one with no upper bound. They also understand that two sets can be added or subtracted to yield a third and that counting forward or backward provides a way to assess the numerical values of these sets. How do children gain this understanding?

Comparing the young schoolchild's number concepts to the core knowledge systems of infants suggests how far children have to go. For infants, small numbers and large numbers are represented differently; for schoolchildren, all natural numbers have the same general properties. Moreover, infants' small-number representations are limited in set size, and their large-number representations are limited in precision, but the schoolchild's number representations show neither limit: The number of individuals in a set can be represented precisely, in principle, and with no upper bound. Finally, infants can perform addition and subtraction on small numbers of objects and they can compare the cardinal values of large sets, but they cannot add or subtract large sets or compare the cardinal values of small numbers of objects; schoolchildren, in contrast, perform additions and numerical comparisons with all set sizes. Studies of infants should lead us to predict, therefore, that developing an understanding of counting and the natural numbers will be difficult for children, and research shows that it is.

Studies by Fuson (1988), Wynn (1990), Griffin and Case (1996), and others reveal that when children first begin to engage in the counting routine, pointing to objects in succession while running through the count list, they have little understanding of what they are doing. For example, Wynn (1990) assessed 2- to 4-year-old children's understanding of the words in their own count lists through a simple task in which she presented a pile of objects and asked children to give her (e.g.) "two fish." The youngest counters correctly gave her one object when asked for one, but they performed at chance, grabbing a handful, when asked for other numbers. (Interestingly, children never gave her just one object when asked for a higher number, suggesting that they understood that the other number words picked out sets larger than one.) About 9 months later, children mastered the meaning of the word *two:* They correctly produced one or two objects when asked for "one fish" or "two fish," respectively, and they grabbed a pile containing more than two objects when asked for any other number. Some 3 months later, on average, children mastered the meaning of the word *three* while continuing to respond at chance for higher numbers. Finally, some time after the acquisition of *three,* children appeared to figure out the workings of the counting routine and the meanings of all the number words. From that point on, children who were asked for any number of objects within their count list would attempt to produce that specific number and would use counting to do so.

The developmental progression observed in preschool children makes sense in the light of the capacities of infants. At the earliest point in the development of number words and counting, I suggest, children learn to relate the word *one* to their core system for representing objects: They learn that *one* applies just in case there's an object in the scene, and it is roughly synonomous with the determiner *a.* About the same time, children learn to relate the other number words to their core system for representing numerosities: They learn that the other number words apply just in case there's a set in the scene, and those words are all roughly synonomous with *some* (see also Bloom and Wynn, 1997). The next and very difficult step requires that children bring their representations of objects and numerosities together. They have to learn that *two* applies just in case there's a set composed of an object and another object. When *two* is mastered, children must learn that *three* also applies to a combination of object and numerosity representations: to a set composed of an object, an object, and an object.

Once this learning is complete, children are in a position to make two general inductions. First, they can discover that the progression from *two* to *three* involves adding one object to the set of objects. Second, they can generalize this discovery to all the number words and infer that each word picks out a set containing one more object than the preceding word. The behavior of number words in natural language syntax, as well as their behavior within the counting routine, may support this generalization (Bloom & Wynn, 1997). The language of number words and the counting routine allow young children to combine their representations of objects as enduring individuals with their representations of numerosities to construct a new system of knowledge of number, in which each distinct number picks out a set of individuals with a distinct cardinal value.

On the view I'm recommending, therefore, children construct the natural number concepts by combining representations from two core systems: the system for representing objects as persisting individuals and the system for representing approximate numerical magnitudes. More specifically, the object system is the source of the child's understanding that number applies to discrete individuals and that numbers can be changed by adding one, and the approximate numerosity system is the source of the child's understanding that number applies to sets and that sets can be compared according to their cardinal values. Number words, the counting routine, and natural language syntax all may support this combination. Children's understanding of the natural number concepts, of the counting routine, and of the counting-based operations of arithmetic may follow from it.

Sources of Mathematical Thinking in Adults

If this picture of core knowledge and cognitive development is correct, then mature number concepts like 5 and 7 and mature arithmetic knowledge like $5 + 7 = 12$ would depend on the orchestration of three systems: a core system for representing small numbers of objects, a core system for representing approximate numerical magnitudes, and the language of number words and verbal counting. All three of these systems should be active in human adults when we represent natural numbers and arithmetic facts. Are they?

First consider the core system for representing objects. For over a hundred years, experimental psychologists have proposed that adults have a special process for representing very small numbers of objects simultaneously and in parallel: a *subitizing* system. Trick and Pylyshyn (1994) have argued that this is the same system that adults use to perform multiple-object tracking, on the basis of evidence that subitizing and multiple-object tracking are influenced by the same stimulus and task variables. Putting those findings together with Scholl and Pylyshyn's (1999) findings described earlier, we have some reason to think that the core system of object representations found in infants plays a role in adults' judgments about small numerosities.

What about the core system for representing large, approximate numerosities? Recent research by Intriligator (1997) provides evidence that large, approximate numerosity discrimination and multiple-object tracking behave very differently in relation to the same stimulus and task variables, providing evidence that the system for representing large, approximate numerosities is distinct from the system for representing small numbers of objects in adults. Nevertheless, a large body of work, beautifully reviewed by Stanislas Dehaene (1997) in his book, *The Number Sense,* provides evidence that the large, approximate numerosity system plays an important role in our mature capacities to compare numbers and perform mental arithmetic. Some of this evidence comes from studies of normal adults who are asked to operate on number words or arabic numerals. When adults are asked to compare two numbers, they show a *distance* effect, responding faster and more accurately when the numbers differ by a larger amount (for example, adults judge that $9 > 5$ faster than they judge that $6 > 5$). When adults are asked to verify whether the answer to an addition problem is correct, they show a *split* effect, spotting incorrect answers faster and more accurately when those answers are more distant from the correct one (e.g., adults judge that $5 + 7$ is not 19 faster than they judge that it is not 13).

Further evidence that the core numerosity system contributes to adults' numerical abilities comes from studies of neurological patients who show two distinct kinds of numerical impairment (see Dehaene & Cohen, 1997, for discussion). One type of patient shows an impaired ability to represent numbers exactly but intact number sense: Such patients may fail to report that the sum of $5 + 7$ is 12 but succeed at reporting that the sum is "close to 13." A different type of patient, typically with damage to the inferior parietal lobes, shows impaired number sense but preservation of many exact arithmetic facts. Such patients can still rattle off facts such as that 8×6 is 48 but no longer show the distance or split effects, and some are unable even to say whether 8 is larger than 6. Although isolated arithmetic facts are preserved, patients with impaired number sense complain of great difficulty with number concepts and mathematics. These difficulties testify vividly to the importance of a sense of approximate numerical magnitudes for our ordinary mathematical abilities (Dehaene, 1997).

These findings suggest that both core representations of objects and core representations of approximate numerical magnitudes play some role in our numerical abilities as adults, but what of the verbal number system? When we represent large exact numerosities, do we use our system of number words to combine the representations delivered by these systems? Two recent lines of research suggest the answer may be yes.

The first research (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999) used two neuroimaging methods to compare the patterns of brain activity elicited in adults by two numerical tasks: a task requiring exact numerical representations and a task requiring approximate numerical representations. In the exact task, participants were given a set of simple addition problems in which they had to select the correct answer from a near miss: for example, $3 + 4 = 7$ rather than 5. In the approximate task, participants were given the same addition problems but were asked to estimate the answer rather than calculate it directly and then to choose a nearby answer from a far miss: for example, $3 + 4$ is about 8 rather than 2. In our first experiment, participants were scanned by means of functional magnetic resonance imaging while performing the exact and approximate tasks in separate blocks, and the activation measured in the two types of blocks was compared, to determine where patterns of neural activity were higher for one task than for the other. The approximate task showed greater bilateral activation throughout the inferior parietal lobes, including both the areas thought to be involved in representations of objects in multiple-object-tracking tasks and those thought to be involved in representations of sets in numerosity discrimination tasks. In contrast, the exact task showed greater activation on the left side of the inferior frontal lobe. The area of activation was one that typically is activated in studies requiring retrieval of well-learned verbal facts and word associations. In a replication of the study using event-related potentials, these two patterns of activation were found to occur very early in processing of the number problems, long before the answers appeared and participants chose a response (Dehaene et al., 1999). These findings suggest that language-dependent systems are involved in representations of exact addition.

Further evidence that core knowledge systems and the language of number words provide the sources of our mathematical thinking comes from a final series of studies on adults, directly inspired by studies of infants and conducted with Sanna Tsivkin and Gail O'Kane. These studies focused on adults who spoke two languages and asked how such adults represent new numerical information that they learn in one of their languages: Is the information represented in a form that is specific to that language, or is it represented independently of language?

We have conducted a number of studies with Russian–English bilinguals (Dehaene et al., 1999; Spelke & Tsivkin, in press),but I will describe just one recently completed study, conducted with Spanish–English bilinguals and focusing on the three hypothesized sources of our number knowledge (O'Kane & Spelke, unpublished data). These bilingual participants learned, over a period of days, to memorize all the information presented in two stories: one story that they learned in English and a different story that they learned in Spanish. Within these stories were facts of various types. Some of the facts had nothing to do with numbers; for example, participants might learn that the heroine of one story loved to wear emeralds and that the hero of the other story loved to eat asparagus. Some of the facts concerned small numbers of objects; for example, that the heroine had two sisters or that the hero had three teachers. Some of the facts concerned large, approximate numerosities; for example, that the hero's mother taught hundreds of students. Finally, some facts concerned large, exact numerosities; for example, that nine treasure chests were lost in a shipwreck. During training, participants learned to identify the correct facts by answering two-choice questions (e.g., what did the heroine love to wear? emeralds vs. rubies). For small-number facts, the two alternatives were both within the small-number system (how many sisters did she have? two vs. three). For the large, approximate numerosity facts, the two alternatives always differed by a 2:1 ratio or more about how many students did the hero's mother teach? hundreds vs. thousands). For the large, exact numerical facts, the alternative was a near miss (how many treasure chests were lost? 9 vs. 8).

After participants had learned all the facts in each of the two stories, one in each of their languages, they were tested on both stories in both their languages. For the nonnumerical facts, performance was independent of language: Participants who learned about emeralds in Spanish were equally fast and accurate at retrieving this information when queried in Spanish and in English. For the facts about small numbers of objects and the facts about large, approximate numerosities, a similar pattern was obtained: Participants retrieved these facts equally well when queried in the two languages. This finding suggests that when participants represent new information about small numbers of objects or about large, approximate numerosities, they are able to represent that information in systems that are independent of language.

When participants were tested on large, exact-number facts, in contrast, they responded more quickly and more accurately when queried in the language in which a fact was learned than when queried in their other language. This finding was obtained both for facts trained in Spanish and for those trained in English, and it suggests that representations of large, exact numerosities depend, in part, on a specific language. All these findings accord with the view that small numbers of objects and large, approximate numerosities are represented by core, language-independent systems. In contrast, large, exact numerosities depend on a combination of representations from core systems, and the language of number words may serve to bind these representations together (see Dehaene et al., 1999, and Spelke & Tsivkin, in press).

Conclusion

My excursion through studies of human infants, nonhuman primates, children learning counting, and mathematically skilled adults centers on one specific and one more general proposal. The specific proposal is that the cognitive functioning of all these disparate groups can be understood, in part, in terms of the same systems of core knowledge. These systems serve to construct abstract representations of basic features of the world, including objects and numerosities, but they are limited in three respects: They are domain specific, task specific, and largely independent of one another. I have focused on one core system for representing objects and a second core system for representing approximate numerical magnitudes. These systems appear to exist both in human infants and in adult monkeys, to dominate young children's earliest attempts to understand number words and the counting routine, and to persist into human adulthood. Moreover, these systems appear to serve as the building blocks for later developing numerical concepts and calculation skills, which children construct and adults deploy by combining representations from the two core systems.

Behind these specific suggestions is a more general proposal. When cognitive and educational psychologists attempt to understand humans' most complex cognitive skills, we should take a broad view and study not only adults who have mastered the skills and children who are acquiring them but also human infants and other animals. Although no young child or nonhuman animal possesses these skills, both exhibit many of the cognitive systems that serve as their building blocks.

The architecture of these systems may be especially amenable to study in infants, where they appear in relatively pure form, and in nonhuman animals, where they can be studied through a rich array of behavioral and physiological methods.

Many of adults' richest and most complex cognitive skills may be assembled from core knowledge systems. For example, our uniquely human patterns of prolific tool use and tool construction may depend on the orchestration of two core systems found in infants: the system for representing objects that I have discussed in this address and a system for representing persons and their goal-directed, intentional actions that has been found both in human infants (e.g., Woodward, 1998) and in nonhuman primates (e.g., Cheney & Seyfarth, 1990). By combining representations from these systems, children may come to view artifacts both as bearers of mechanical properties and as products of human intentions: representations that become well established during the preschool years (Bloom, 1996; Kelemen, 1999).

As a second example, I've argued elsewhere that our uniquely human propensity to navigate flexibly may result from the orchestration of the system for representing objects with a core system for representing the geometry of the spatial layout (e.g., Hermer and Spelke, 1996; Hermer-Vazquez, Spelke, & Katsnelson, 1999). By combining representations from these systems, human children may come to navigate not only as other animals do, by maintaining their sense of orientation in a geocentric representation of the permanent environment, but in more flexible ways that allow us to get from place to place even when our sense of orientation is lost or when we find ourselves in novel surroundings. In these cases and others, human children and adults may gain new abilities not by creating those abilities out of whole cloth, but by bringing together building-block representational systems that have existed in us since infancy. By shedding light on those systems, studies of human infants may contribute to understanding of some of the highest achievements of human adults.

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