

Painting by Renée Jorgensen Bolinger

Gottlob Frege On Sense and Reference

Gottlob Frege

(1848-1925)

Life's work: *logicism*

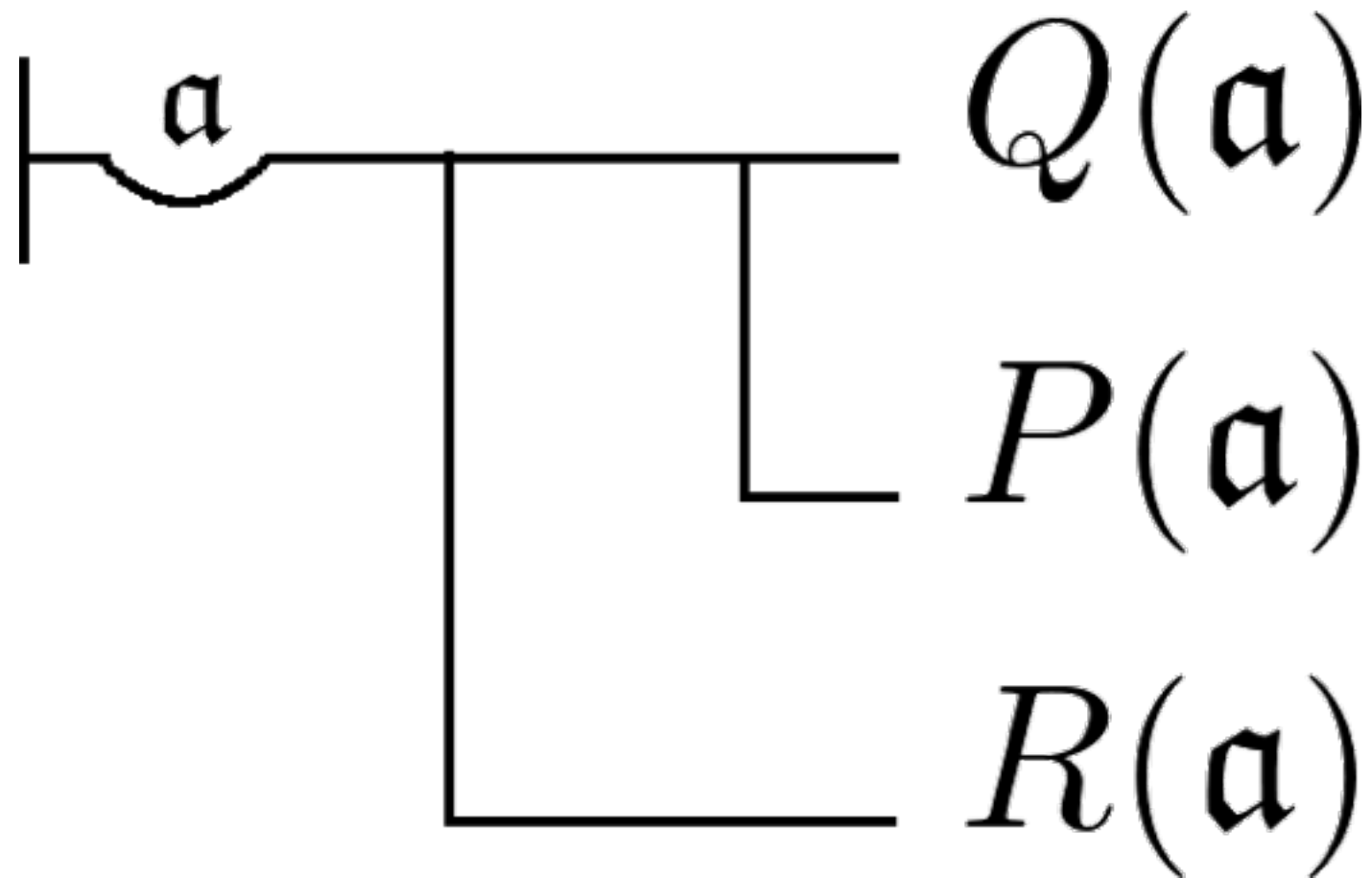
(the reduction of arithmetic to logic).

This entailed:

- Inventing (discovering?) modern logic, including quantification, variables, etc.
- Investigating the properties of the language in which this logic was spelled out.

Step 1: *Begriffsschrift* (1879)

...in which Frege comes up with "classical" logic.



$$(\forall x)(Rx \supset (Px \supset Qx))$$

Step 2: *Die Grundlagen der Arithmetik* (1884)

...in which Frege proposes his definition of number.



$3 =_{df}$ the concept true of all concepts that are true of exactly three things.

Step 3: *Die Grundgesetze der Arithmetik* (1893–1903)

...in which Frege attempts to derive arithmetic
from logic



Step 2.5: Three Philosophical Papers

Funktion und Begriff (1891)

('Concept and Object')

...in which Frege proposes that concepts (the meanings of predicates) should be thought of as *functions*.

Über Begriff und Gegenstand (1892)

('Concept and Object')

...in which Frege argues that concepts/functions differ from objects in that they're "unsaturated".

Über Sinn und Bedeutung (1892)

('On Sense and Reference')

...in which Frege elaborates his theory of words' meanings.

Über Sinn und Bedeutung
(‘On Sense and Reference’)

Each expression has (at least) the following two semantic properties:

- **sense** (*Sinn*)
- **reference** (*Bedeutung*)

* (Frege also mentions a third kind in passing: **coloring** (*Färbung*)).

"It is natural, now, to think of there being connected with a sign (name, combination of words, letter), besides that to which the sign refers, which may be called the reference of the sign, also what I should like to call the sense of the sign, wherein the mode of presentation is contained."

Frege's Puzzle:

Compare:

(1) Hesperus is identical to Hesperus.

(2) Hesperus is identical to Phosphorus.

and:

(3) The morning star is identical to the morning star.

(4) The morning star is identical to the evening star.

Frege's Puzzle (in general):

Identity statements of the form ' $a=a$ ' are trivial, but those of the form ' $a=b$ ' are nontrivial. We have to believe the first kind but not the second kind.

They differ in *cognitive value*.

If the meaning of a name (a proper name or a description) is just its referent, we can't explain this.

Frege's Solution:

"If we found " $a=a$ " and " $a=b$ " to have different cognitive values, the explanation is that for the purpose of knowledge, the sense of the sentence, viz., the thought expressed by it, is no less relevant than its referent, i.e., its truth value. If now $a=b$, then indeed the referent of " b " is the same as that of " a ," and hence the truth value of " $a=b$ " is the same as that of " $a=a$." In spite of this, the sense of " b " may differ from that of " a ," and thereby the sense expressed in " $a=b$ " differs from that of " $a=a$." In that case the two sentences do not have the same cognitive value."

An expression's sense plays several roles:

- It is a mode of presentation of the expression's referent
- It determines the expression's referent.
- It is the what is grasped by a speaker who understands the expression
- They are what is communicated.
- It is responsible for the expression's "cognitive value".
- It is the referent of the expression in "indirect" (*ungerade*) linguistic contexts.

Frege's Anti-Psychologism

"The reference and sense of a sign are to be distinguished from the associated idea. ... The idea is subjective: one man's idea is not another. ... This constitutes an essential difference between the idea and the sign's sense, which may be the common property of many and therefore is not a part of a mode of the individual mind." (8th paragraph)

Frege's Anti-Psychologism

"By a thought I understand not the subjective performance of thinking but its objective content, which is capable of being the common property of several thinkers." (fn.7)

Sentences Refer to Truth Values

Why does Frege think this?

Sentences Refer to Truth Values

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For the best explanation I've ever read, see Stephen Neale, *Facing Facts* (OUP, 2001), Ch.3, "Frege: Truth and Composition" (email me for a copy).

<p>SIGN</p> <p>a linguistic expression</p>	<p>NAME</p> <p>'Aristotle'</p>	<p>PREDICATE</p> <p>'...is wise'</p>	<p>SENTENCE</p> <p>'Aristotle is wise.'</p>
<p>SENSE</p> <p>the abstract object expressed by the sign</p>	<p>(INDIVIDUAL) CONCEPT</p> <p>a mode of presentation of an individual</p> <p><i>the student of Plato and teacher of Alexander</i></p>	<p>(GENERAL) CONCEPT</p> <p>function from individual senses to thoughts</p> <p><i>the concept of wisdom</i></p>	<p>THOUGHT</p> <p>an entity with truth conditions that can be grasped by different people</p> <p><i>the thought that Aristotle is wise</i></p>
<p>REFERENCE</p> <p>what the sign stands for or designates</p>	<p>AN INDIVIDUAL</p> <p><i>Aristotle himself</i></p>	<p>PROPERTY</p> <p>function from things to truth values</p> <p><i>the function that outputs the true if you give it a wise thing and false otherwise</i></p>	<p>TRUTH VALUE</p> <p>either The True or The False</p> <p>in this case, The True</p>

Functions and Compositionality

Frege's ideas allow for a very systematic approach to compositionality.

- Singular terms refer to entities.
Their semantic type is e .
- Sentences refer to truth values.
Their semantic type is t .
- Other expressions' types can be understood as functions defined in terms of these two types.
 - E.g., predicates ("...is wise") are type $\langle e \rightarrow t \rangle$: their referents are functions that take entities as inputs and give back truth values as outputs.

Sense as Indirect Reference

We can normally substitute co-referring expressions without changing the reference of expressions in which they appear.

e.g.:

Hesperus is identical to Venus.

Phosphorus is identical to Venus.

Sense as Indirect Reference

Some syntactic positions are non-extensional: they don't allow substitution of co-referring expressions.

Lucy believes that **Hesperus** is identical to Venus.

Lucy believes that **Phosphorus** is identical to Venus.

Sense as Indirect Reference

Frege's solution to this problem is that expressions in these "oblique" (*ungerade*) contexts refer to their usual senses.

Lucy believes that **Hesperus** is identical to Venus.

Lucy believes that **Phosphorus** is identical to Venus.

Non-Extensional Contexts

Anything embedded under a "propositional attitude verb"

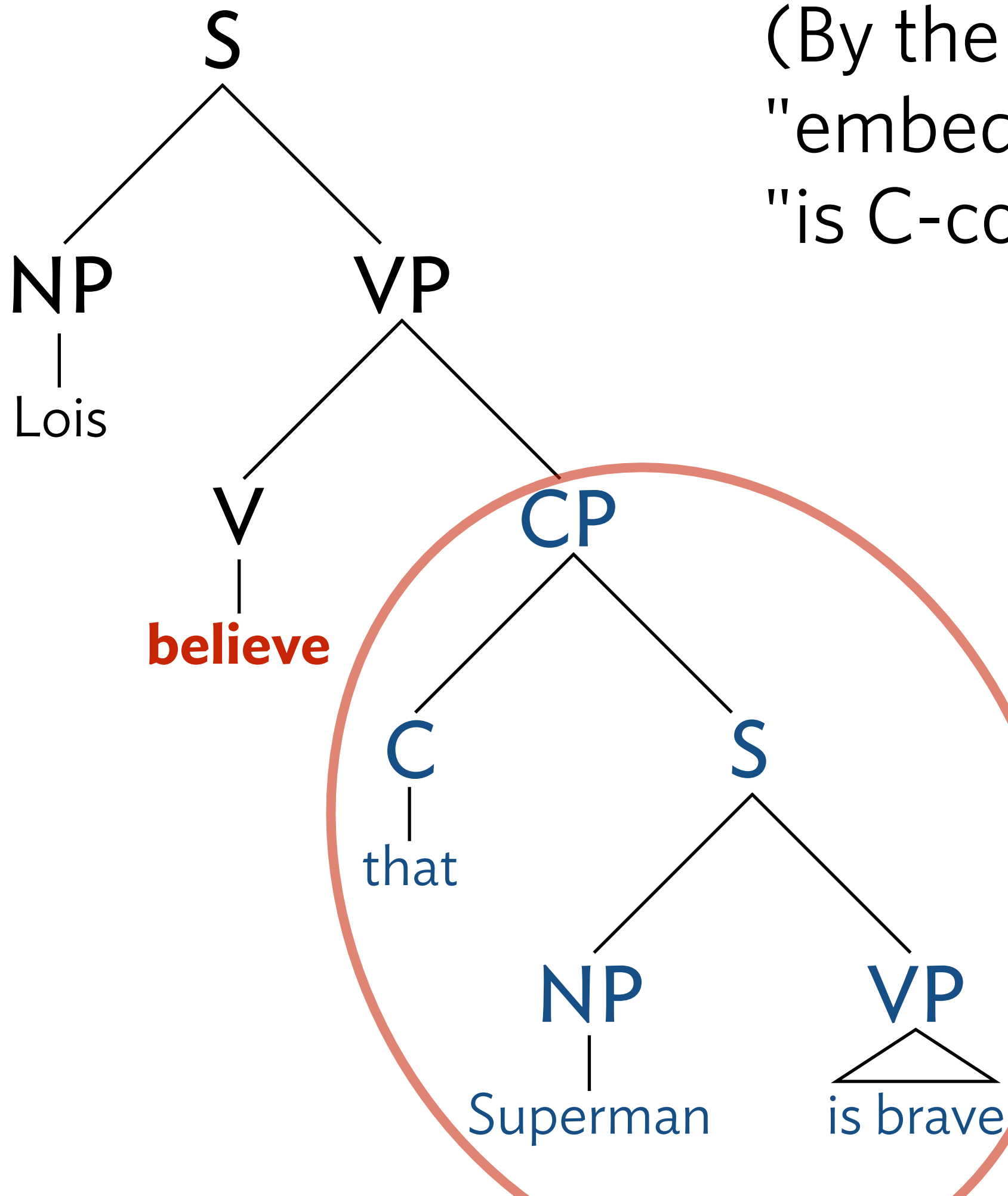
Lois **believes** that Superman is not Clark Kent.

Lois **desires** superman.

Lois **said** that superman is brave.

Lois went outside **in order to** see Superman.

(By the way:
"embedded under" =*df.*
"is C-commanded by".)



Non-Extensional Contexts

Anything embedded under a modal

Necessarily, earth is identical to the third planet from the sun.

It could have been the case that Aristotle was not the student of Plato.

Some "Simple Sentences"

Clark Kent went into the phone booth, and
Superman came out.

It's not obvious what the explanation for these would be for Frege. For a book-length discussion of the problems they raise, see Jennifer Saul, *Simple Sentences, Substitution, and Intuitions*, OUP 2007.